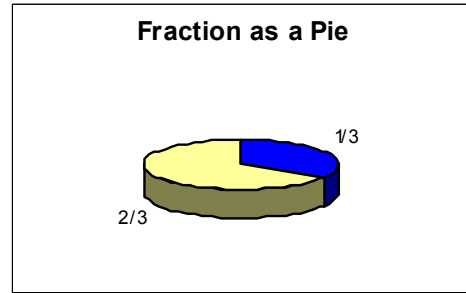


# FRACTIONS

Fractions are usually pictured as a part of a whole pie, as shown at the right. The parts of a fraction are:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{part}}{\text{whole}}$$

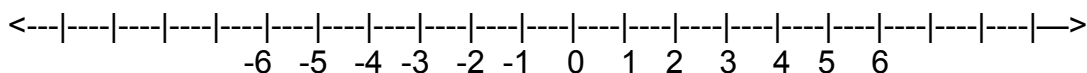


The rules for fraction operations are:

Rule	Example
<p><b>1. Addition/Subtraction:</b> Find the lowest common denominator; rewrite fractions with LCD; add or subtract; reduce.</p>	$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$ $\frac{7}{15} - \frac{3}{10} = \frac{14}{30} - \frac{9}{30} = \frac{5}{30} = \frac{1}{6}$
<p><b>2. Multiplication:</b> Cancel first; multiply across.</p>	$\frac{3}{10} \cdot \frac{5}{9} = \frac{1}{10} \cdot \frac{5}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$ $2\frac{1}{2} \cdot 3\frac{1}{4} = \frac{5}{2} \cdot \frac{13}{4} = \frac{65}{8} = 8\frac{1}{8}$
<p><b>3. Division:</b> Invert the second fraction; multiply (using Rule 2 above).</p>	$\frac{2}{3} \div \frac{5}{4} = \frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$ $2\frac{1}{4} \div 1\frac{1}{2} = \frac{9}{4} \div \frac{3}{2} = \frac{9}{4} \cdot \frac{2}{3} = \frac{3}{2} = 1\frac{1}{2}$

## INTEGERS

The *integers* are basically the positive and negative whole numbers, as shown on the number line below.



The *absolute value* of an integer is its distance from 0; for instance  $|7| = 7$  and  $|-4| = 4$ .

The following are rules for the operations with integers.

Rule	Example
<p><b>1a. Addition (same sign):</b></p> <p>Add the absolute values; give answer the common sign.</p>	<p><math>7 + 3 = 10</math></p> <p><math>-2 + (-5) = -7</math></p>
<p><b>1b. Addition (opposite signs):</b></p> <p>Subtract absolute values; give answer the sign of the larger absolute value.</p>	<p><math>7 + (-4) = 3</math></p> <p><math>-2 + 9 = 7</math></p> <p><math>-8 + 3 = -5</math></p>
<p><b>2. Subtraction:</b></p> <p>Change the sign of the second number; add using Rules 1a and 1b above.</p>	<p><math>7 - (-2) = 7 + 2 = 9</math></p> <p><math>-2 - 5 = -2 + (-5) = -7</math></p> <p><math>-10 - (-4) = -10 + 4 = -6</math></p>
<p><b>3. Multiplication:</b></p> <p>Multiply the absolute values.</p> <p>a) If same sign, answer is positive.</p> <p>b) If opposite sign, answer is negative.</p>	<p><math>3 \cdot 5 = 15</math></p> <p><math>(-2) \cdot (-4) = 8</math></p> <p><math>7 \cdot (-3) = -21</math></p> <p><math>-9 \cdot (2) = -18</math></p>
<p><b>4. Division:</b></p> <p>Divide the absolute values.</p> <p>a) If same sign, answer is positive.</p> <p>b) If opposite sign, answer is negative.</p>	<p><math>20 \div 4 = 5</math></p> <p><math>(-15) \div (-5) = 3</math></p> <p><math>40 \div (-10) = -4</math></p> <p><math>-36 \div 4 = -9</math></p>

# PERCENTS

*Percent means out of 100.* Percents and decimals are converted to each other by moving the decimal point.

*Percent to decimal:* Move the decimal point two places left.

**Examples:** 45% → 0.45;      0.2% → 0.002;      120% → 1.2

*Decimal to percent:* Move the decimal point two places right.

**Examples:** 0.713 → 71.3%;      0.0003 → 0.03%      1.75 → 175%

Percent problems come in two main types: *Whole-Parts Problems* and *Change Problems*.

1. *Whole-Parts Problems*      Here we have the formula with three quantities:

$$A = P \cdot B$$

where A = Amount (part)  
P = Percent (same units as Amount)  
B = Base (whole, total)

In words, the formula says “The part (A) is some percent (P) of the whole (B).”

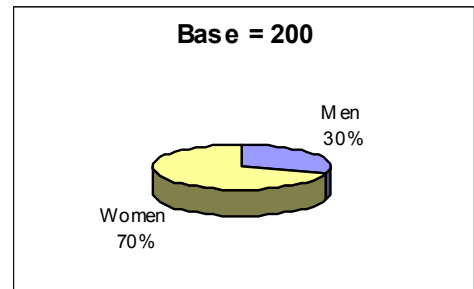
**Example**      A church of 200 people is 30% men. How many men are in the church.

**Solution**      The pie at the right shows this situation. The whole (base) is 200. The percent of men is 30% (= 0.30), and we want the amount of men.

$$A = P \cdot B \quad \text{Basic formula}$$

$$A = (0.30)(200) \quad \text{Substitute}$$

$$A = 60 \text{ men} \quad \text{Multiply}$$



Also, we know the church is 70% women, or 140 women.

2. **Change Problems** These are very similar to whole-part problems, with the same formula.

$$A = P \cdot B$$

where A = Amount (change)  
 P = Percent  
 B = Base (original)

In words, this formula says, "The change (A) is some percent (P) of the original (B)."

**Example** A coat costs \$300, and a store gives a 40% discount. What is the new price?

**Solution** Here the base B is the original price \$300. The percent P is 40% (= 0.40) *discount*. The formula will give the amount A of the *discount*.

$$A = P \cdot B \quad \text{Basic formula}$$

$$A = (0.40)(300) \quad \text{Substitute}$$

$$A = \$120 \text{ discount} \quad \text{Simplify}$$

This is the *discount*, not the new price. To find the new price, we have to subtract: \$300 - \$120 = \$180.

**Example** At Chipmunk University, there are 1200 freshmen, 1100 sophomores, 900 juniors, and 800 seniors. What percent of the school are the freshmen?

**Solution** This is a whole-parts problem. We have to add all the students to get the whole (B). Thus, B = 1200 + 1100 + 900 + 800 = 4000. The part (A) is 1200 freshmen. The unknown is P (percent of freshmen).

$$A = P \cdot B \quad \text{Basic formula}$$

$$1200 = P \cdot 4000 \quad \text{Substitute}$$

$$1200/4000 = P \quad \text{Divide by 4000}$$

$$0.30 = P \quad \text{Divide}$$

$$30\% = P \quad \text{Convert to percent}$$

## RATIO AND PROPORTION

A *ratio* is a comparison of two quantities by division. Often we see the words *per* or *to* used to indicate a ratio.

**Examples:** miles per gallon – 300 miles ÷ 10 gallons = 30 miles per gallon  
men to women – 50 men ÷ 100 women = 0.5 men-to-women

A *proportion* is two equal ratios. We use these to convert units, compare similar triangles, determine fair shares, etc.

**Example** One kilogram is 2.2 pounds. A truck weighs 6600 pounds. How many kilograms is this?

**Solution** We set up two equal kilogram-to-pound ratios. Then we fill in the given information, letting  $x$  = weight of the truck in kilograms.

$$\frac{\text{kilogram}}{\text{pounds}} = \frac{\text{kilogram}}{\text{pounds}} \quad \text{Set up proportion}$$

$$\frac{1 \text{ kilogram}}{2.2 \text{ pounds}} = \frac{x \text{ kilogram}}{6600 \text{ pounds}} \quad \text{Fill in information}$$

$$6600(1) = 2.2x \quad \text{Cross multiply}$$

$$3000 = x \quad \text{Divide by 2.2}$$

**Example** Bob puts up \$5 and Jane puts up \$10, and they buy a bunch of lottery tickets. One of their tickets wins \$300,000. What is Bob's share?

**Solution** Here we set up the ratios of Bob-to-Total. Let  $x$  = Bob's share.

$$\frac{\text{Bob}}{\text{Total}} = \frac{\text{Bob}}{\text{Total}} \quad \text{Set up proportion}$$

$$\frac{\$5 \text{ Bob}}{\$15 \text{ Total}} = \frac{x \text{ (Bob)}}{\$300,000 \text{ (Total)}} \quad \text{Fill in information}$$

$$5(300,000) = 15x \quad \text{Cross multiply}$$

$$100,000 = x \quad \text{Divide by 15}$$

## SOLVING EQUATIONS

The key to solving equations is getting  $x$  (the unknown) by itself. We simplify both sides of the equation as much as possible. Then we move all the  $x$ -terms to one side of the equations, and all the other terms to the other side. Also, whatever we do to the left side, we do to the right side.

**Example 1**       $2x = 10$

$$x = 5 \qquad \text{Divide by 2}$$

**Example 2**       $3x - 7 = 23$

$$3x = 30 \qquad \text{Add 7}$$

$$x = 10 \qquad \text{Divide by 3}$$

**Example 3**       $3x + 4 = 5x - 22$

$$4 = 2x - 22 \qquad \text{Subtract } 3x$$

$$26 = 2x \qquad \text{Add 22}$$

$$13 = x \qquad \text{Divide by 2}$$

**Example 4**       $\frac{x}{100} = \frac{3}{20}$

$$20x = 300 \qquad \text{Cross multiply}$$

$$x = 15 \qquad \text{Divide by 20}$$

**Example 5**       $2x^2 - 50 = 0$

$$2x^2 = 50 \qquad \text{Add 50}$$

$$x^2 = 25 \qquad \text{Divide by 2}$$

$$x = 5 \qquad \text{Square root}$$

**Example 6**       $y = 4x + 13$

$$y = 6x - 7$$

$$y = 4x + 13 = 6x - 7$$

$$13 = 2x - 7$$

$$20 = 2x$$

$$10 = x$$

*Set y-values equal to each other*

*Subtract 4x*

*Add 7*

*Divide by 2*

## TRANSLATING ENGLISH INTO MATHEMATICS

To solve many of the problems, it is necessary to translate English terms and sentences into mathematical symbols and equations. The following are some words and their mathematical equivalents.

English	Mathematics
and, plus, total, more than, together	+
difference, less, less than, minus, take away, how much more	-
times, product, of, each ... at	·
quotient, ratio, out of, per, to	÷
what, some number	x
is, will be, should be, results in	=
percent	decimal

Here are some examples of these.

English	Mathematics
Two <u>more than</u> some number	$2 + x$
Seven <u>less than</u> the height	$h - 7$
20 tickets <u>at</u> \$10 <u>each</u>	$20 \cdot 10$
200 miles <u>per</u> 10 gallons	$200 \div 10$
Five <u>less than</u> 4 <u>times</u> Joe's age	$4x - 5$
The length <u>is</u> 8 <u>more than</u> the width	$L = 8 + W$
Fifteen <u>is</u> 10% <u>of</u> what number?	$15 = 0.10 \cdot x$
Four <u>out of</u> five doctors	$4/5$
What <u>is</u> 20% <u>of</u> 6000?	$x = (0.20) \cdot 6000$
The <u>difference between</u> Sue's age and Bob's age <u>is</u> seven	$S - B = 7$
Fifty <u>is</u> what percent <u>of</u> 400?	$50 = x \cdot 400$

Al <u>and</u> Bob <u>weigh</u> 410 pounds together	$A + B = 410$
One-third <u>of</u> a number <u>is</u> 40	$\frac{1}{3} \cdot x = 40$
The cost <u>is</u> \$10 <u>plus</u> \$0.25 a minute	$C = 10 + 0.25x$
The number of people in Room 1 <u>is</u> 8 <u>more than</u> in Room 2.	$R_1 = 8 + R_2$

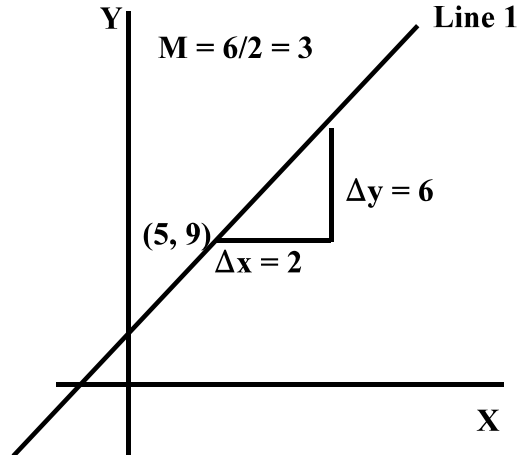
## EQUATION OF A LINE

When put on an x-y grid, a straight line has a *slope*  $m$  that measures its steepness.

The slope  $m$  is given in several equivalent ways. (We use  $\Delta$  to mean “change in”.)

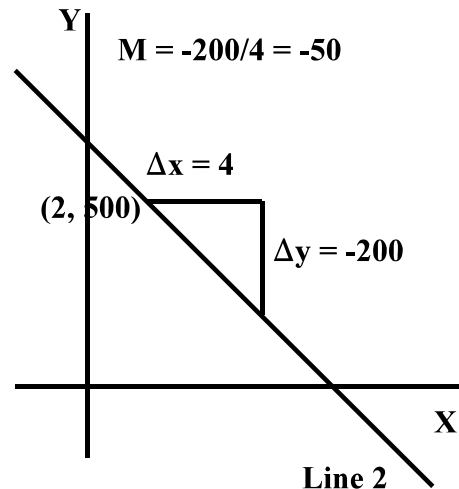
$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

Line 1 at the right, the rise (change in  $y$ ) is 6; the run (change in  $x$ ) is 2. So, the slope  $m = 6/2 = 3$ . Basically, this says that the line is rising up 3 times faster than it is going to the right.



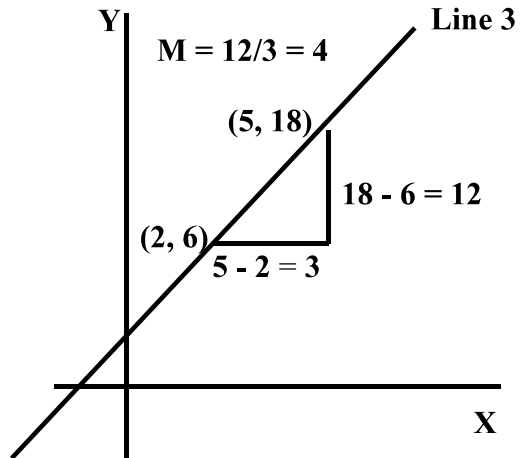
Line 2 at the right is falling (as we go left to right), so the slope should be negative. In this case we have

$$m = \frac{\text{rise}}{\text{run}} = \frac{-200}{4} = -50$$



We can find the rise and run by subtracting the y- and x-coordinates of the two points. Be careful always to use the order (for example, right-point coordinates - left-point coordinates). In Line 3, we have

$$m = \frac{\text{rise}}{\text{run}} = \frac{18 - 6}{5 - 2} = \frac{12}{3} = 4$$



Once we have the slope and a point on a line, we can find its equation using the *point-slope formula*:

$$y - y_1 = m(x - x_1)$$

where  $m$  is the slope and  $(x_1, y_1)$  is a given point of the line.

Using the three lines above as examples, we have the following.

<b>Line 1:</b>	$m = 3; (x_1, y_1) = (5, 9)$	<i>Identify point and slope</i>
	$y - y_1 = m(x - x_1)$	<i>Point-slope formula</i>
	$y - 9 = 3(x - 5)$	<i>Substitute</i>
	$y - 9 = 3x - 15$	<i>Simplify</i>
	$y = 3x - 6$	<i>Add 9</i>

So, the equation of Line 1 is  $y = 3x - 6$ .

<b>Line 2:</b>	$m = -50; (x_1, y_1) = (2, 500)$	<i>Identify point and slope</i>
	$y - y_1 = m(x - x_1)$	<i>Point-slope formula</i>
	$y - 500 = -50(x - 2)$	<i>Substitute</i>
	$y - 500 = -50x + 100$	<i>Simplify</i>
	$y = -50x + 600$	<i>Add 500</i>

So, the equation of Line 2 is  $y = -50x + 600$ .

<b>Line 3:</b>	$m = 4; (x_1, y_1) = (2, 6)$	<i>Identify point and slope</i>
	$y - y_1 = m(x - x_1)$	<i>Point-slope formula</i>
	$y - 6 = 4(x - 2)$	<i>Substitute</i>
	$y - 6 = 4x - 8$	<i>Simplify</i>
	$y = 4x - 2$	<i>Add 6 [6 + (-8) = -2]</i>

So, the equation of Line 3 is  $y = 4x - 2$ .

## STATISTICS

The *mean* or *average* of a set of numbers is found by adding them and dividing by the number of values. The *median* of a set of numbers is the number which half are above and half are below.

**Example** Find the mean and median of {10, 13, 20, 27, and 85}.

**Solution** The mean (or average) is the sum divided by 5.

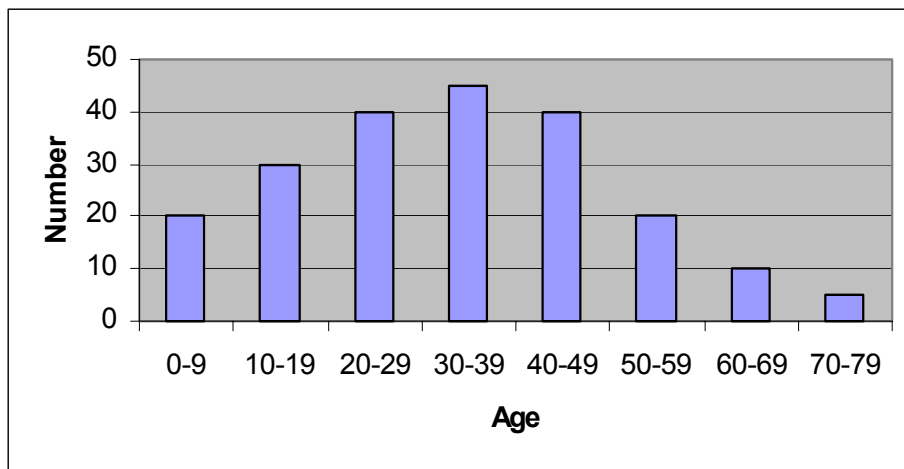
$$\text{mean} = \frac{10 + 13 + 20 + 27 + 85}{5} = \frac{155}{5} = 31$$

The median is middle number as the numbers are ordered, smallest to largest. Here, the median is 20, since half the number below 20, and half are above.

If there are a lot of values or data, they might be grouped in a table, which can then be pictured as a *histogram*.

**Example** The data below (ages of people in a church) is groups of people in intervals of 10: 0 to 9, 10 to 19, 20 to 29, etc. Then it is drawn as a histogram.

Age	Number
0 - 9	20
10 - 19	30
20 - 29	40
30 - 39	45
40 - 49	40
50 - 59	20
60 - 69	10
70 - 79	5



## EXPONENTS AND SCIENTIFIC NOTATIONS

*Exponents* are repeated multiplication.

$$x^5 = x \times x \times x \times x \times x = (5 \text{ factors of } x)$$

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16 \text{ (4 factors of 2)}$$

The rules for terms in exponent form are:

Rule	Examples
$a^m a^n = a^{m+n}$	$x^4 x^5 = x^9$ $10^7 \cdot 10^4 = 10^{11}$
$a^m / a^n = a^{m-n}$	$y^8 / y^5 = y^3$ $2^{10} / 2^4 = 2^6$
$(a^r)^s = a^{rs}$	$(z^3)^4 = z^{12}$ $(10^2)^3 = 10^6$
$(ab)^m = a^m b^m$	$(xy)^7 = x^7 y^7$ $(2a)^3 = 2^3 a^3 = 8a^3$

Very big numbers are written in *scientific notation* by moving the decimal point until the number is between 1 and 10. The number of places moved left is the power of 10.

**Example**      $60000 = 6,0000 = 6.0 \times 10^4$   
<-----  
4 places

$450,000,000 = 4,50,000,000 = 4.5 \times 10^8$   
<-----  
8 places

$2000 = 2,000 = 2 \times 10^3$   
<---  
3 places

We can multiply numbers in scientific notation, as follows:

$$(6 \times 10^4) \times (2 \times 10^3) = 12 \times 10^7 = (1.2 \times 10^1) \times 10^7 \quad \text{Since } 12 = 1.2 \times 10^1$$

$$= 1.2 \times 10^8$$

## POLYNOMIALS

*Polynomials* are expressions involving powers of  $x$ , such as

$$10x^3 - 7x^2 + 12x + 17$$

The numbers 10, -7, 12, and 17 are called *coefficients*; the terms  $x^3$ ,  $x^2$ , and  $x$  are called *powers of  $x$* . The operations on polynomials are as follows:

Rule	Example
<b>1. Addition</b> Line up common terms; add coefficients.	$\begin{array}{r} 7x^3 + 5x^2 \quad \quad + 12 \\ 9x^3 \quad \quad \quad - 4x - 3 \\ \hline 16x^3 + 5x^2 - 4x + 9 \end{array}$
<b>2. Subtraction</b> Change <u>all</u> the signs in the second polynomial; add using Rule 1 above.	$\begin{aligned} & (8x^2 - 5x + 7) - (3x^2 + 7x - 4) \\ = & (8x^2 - 5x + 7) + (-3x^2 - 7x + 4) \\ & \begin{array}{r} ( 8x^2 - 5x + 7) \\ \underline{(-3x^2 - 7x + 4)} \\ 5x^2 - 12x + 11 \end{array} \end{aligned}$
<b>3. Multiplication</b> Special products:  $(a+b)(c+d) = ac + ad + bc + bd$  $(a + b)(a - b) = a^2 - b^2$  $(a + b)^2 = a^2 + 2ab + b^2$	$\begin{aligned} (6x+7)(4x-2) &= 24x^2 - 12x + 28x - 14 \\ &= 24x^2 + 16x - 14 \\ \\ (3x + 4)(3x - 4) &= 9x^2 - 16 \\ \\ (3x - 5y)^2 &= 9x^2 - 30xy + 25y^2 \end{aligned}$

## FACTORING

*Factoring* is the opposite of multiplying. You can split a whole number or polynomials into factors. With polynomials, we factor using the same rules for multiplying in reverse.

$$30 = 6 \cdot 5 = 2 \cdot 3 \cdot 5$$

$$75 = 5 \cdot 15 = 5 \cdot 3 \cdot 5$$

$$x^2 - 4 = (x + 2)(x - 2)$$

$$9x^2 - 25 = (3x + 5)(3x - 5)$$

$$x^2 + 13x + 40 = (x + 8)(x + 5)$$

$$x^2 - 2x - 15 = (x - 5)(x + 3)$$

$$4x^2 + 12x + 9 = (2x + 3)(2x + 3) = (2x + 3)^2$$

Many quadratic equations (second degree – with an  $x^2$ ) can be solved by factoring (and some cannot).

**Example:** Solve  $x^2 - 3x - 28 = 0$

**Solution:** We factor the left side and set each factor equal to 0.

$$x^2 - 3x - 28 = 0$$

*Given equation*

$$(x - 7)(x + 4) = 0$$

*Factor*

$$x - 7 = 0 \quad \text{and} \quad x + 4 = 0$$

*Set each factor = 0*

$$x = 7 \quad \text{and} \quad x = -4$$

*Solve each equation*

## Rational Expressions (Algebraic Fractions)

The following are the rules for the operations on algebraic fractions. Notice that the rules are very similar to the rules for the operations on number fractions.

Rule	Example
<b>1. Reducing</b> Factor and cancel common terms.	$\frac{x^2 - x - 6}{x^2 + x - 2} = \frac{(x-3)(x+2)}{(x-1)(x+2)}$ $= \frac{x-3}{x-1}$
<b>2. Addition/Subtraction</b> Rewrite with an LCD; add/subtract numerators.	$\frac{1}{a} + \frac{1}{b} = \frac{b}{ab} + \frac{a}{ab} = \frac{b+a}{ab}$ $\frac{5}{x-2} - \frac{3}{x+6} = \frac{5(x+6)}{(x-2)(x+6)} + \frac{-3(x-2)}{(x-2)(x+6)}$ $= \frac{5x+30-3x+6}{(x-2)(x+6)} = \frac{2x+36}{(x-2)(x+6)}$
<b>3. Multiplication</b> Multiply across; cancel.	$\frac{6a}{7b^2} \cdot \frac{1}{4a^4} = \frac{6a}{28a^4b^2} = \frac{3}{14a^3b^2}$
<b>4. Division</b> Invert and multiply.	$\frac{2}{a+b} \div \frac{5}{x+y} = \frac{2}{a+b} \cdot \frac{x+y}{5} = \frac{2(x+y)}{5(a+b)}$
<b>5. Solving equations with fractions</b> Multiply both sides by the LCD; solve.	$\frac{1}{2} - \frac{1}{x} = \frac{1}{3}$ $6x \left[ \frac{1}{2} - \frac{1}{x} \right] = 6x \left[ \frac{1}{3} \right] \quad \text{Multiply by LCD, } 6x$ $3x - 6 = 2x \quad \text{Distributive Law}$ $x = 6 \quad \text{Solve}$

## RADICALS

Radicals like  $\sqrt{N}$  tell what number times itself gives N. For example,

$$\sqrt{4} = 2, \quad \sqrt{9} = 3, \quad \sqrt{16} = 4, \quad \sqrt{25} = 5, \quad \text{etc.}$$

Radicals can be simplified by factoring out perfect squares.

**Example**  $\sqrt{300} = \sqrt{100}\sqrt{3} = 10\sqrt{3}$

$$\sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

$$\begin{aligned}\sqrt{20} + \sqrt{80} &= \sqrt{4}\sqrt{5} + \sqrt{16}\sqrt{5} \\ &= 2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5}\end{aligned}$$

To simplify fractions with radicals in the denominator, multiply the top and bottom by the conjugate of the radical expression.

**Example**  $\frac{4}{6-\sqrt{5}} = \frac{4}{6-\sqrt{5}} \cdot \frac{6+\sqrt{5}}{6+\sqrt{5}} = \frac{24+4\sqrt{5}}{36-5} = \frac{24+4\sqrt{5}}{31}$

To solve an equation with radicals, square both sides, solve, and check the answer.

**Example** Solve  $\sqrt{2x-1} = 3$

$$2x - 1 = 9 \quad \text{Square both sides}$$

$$x = 5 \quad \text{Solve}$$

The cube root  $\sqrt[3]{N}$  means the number whose cube is N. For example,

$$\sqrt[3]{8} = 2, \quad \sqrt[3]{27} = 3, \quad \sqrt[3]{64} = 4, \quad \text{etc.}$$

Radicals can be written with fraction exponents; for instance,

$$\sqrt{x} = x^{1/2}, \quad \sqrt[3]{y} = y^{1/3}, \quad \sqrt[4]{z} = z^{1/4}, \quad \text{etc.}$$

**Examples:**  $25^{3/2} = (\sqrt{25})^3 = 5^3 = 125$

$$8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = 16$$